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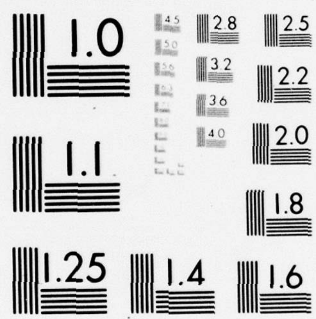

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Combination of Satellite Altimetric Data in the Short Arc Mode and Gravity Anomaly Data

GEORGE HADGIGEORGE
GEORGES BLAHA

3 June 1977

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FOR THE COMMANDER

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The Air Force Geophysics Laboratory (AFGL) computer program SAGG (Satellite Altimetry and Ground Gravity) has been used to determine the global geoid and the earth's gravity field, based on the combination of altimetric observations and gravity anomalies. A typical feature of SAGG is the simultaneous recovery of the orbit parameters and the spherical harmonic potential coefficients. The short arc adjustment mode makes these determinations possible without the requirement of highly precise reference orbits. In this mode, → next page

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20. Abstract (Continued)

the state vector components are subject to adjustment and represent, in fact, a set of independent weighted parameters. Orbits good to approximately 20 m are adequate for precise reductions. Altimetric data processed by SAGG was gathered by the GEOS-3 satellite over adjacent portions of the Indian and South Pacific Oceans and a portion of the North Atlantic; gravity anomaly data is represented by mean anomalies from over 2200 1° × 1° geographic blocks. The recovered geoid over most of the globe shows good agreement with gravimetric geoids. This is especially true of the areas covered by GEOS-3 when compared with the earlier reported results of the AFGL computer program SARRA (Short Arc Reduction of Radar Altimetry).

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Contents

Illustrations

Combination of Satellite Altimetric Data in the Short Arc Mode and Gravity Anomaly Data

1. INTRODUCTION

The short arc adjustment mode of satellite altimetry makes a determination of the geoid surface possible without the requirement of highly precise reference orbits. In fact, approximate values of the state vector parameters with some fairly large a-priori standard deviations are sufficient. The observations gathered over the oceanic regions represent the distances from a satellite to the geoid. For many purposes, the geoid surface is assumed to coincide with the surface of oceans. For a good determination of the geoid everywhere, it is necessary to combine satellite altimetric data with some other data gathered over the continental regions, such as gravity observations. In this paper, satellite altimetry data whose nature will be described later is combined with mean gravity anomalies representing over 2200 $1^{\circ} \times 1^{\circ}$ geographic blocks covering many continental as well as oceanic regions.

The pertinent analysis of the (short arc) satellite altimetry model and the gravity anomaly model will be preceded by a few explanatory remarks with regard to the former model, the more complex of the two. The parameters in this model are divided into two basic groups: (1) the earth potential coefficients which are the conventional C's and S's of the spherical harmonic expansion including the "central term" r_o, and (2) six state vector components for each short arc. The

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parameters in the first group may be considered completely free to adjust, except for the five "forbidden" coefficients, C_{10}, C_{11}, S_{11}, C_{21}, S_{21}, which are usually set to zero and held at that value. This implies that the origin of the Cartesian coordinate system is identical with the earth's center of mass and that the z-axis coincides with the earth's axis of rotation. The parameters in the second group, together with their a-priori error characteristics, are assumed to be supplied from an independent source. They may be given in one inertial coordinate system for all arcs, or in a chosen inertial coordinate system which may vary from arc to arc. For convenience, the parameters may coincide with the earth fixed (E. F.) coordinate system at the epoch time, or in the E. F. system itself. We choose this last possibility which leads to a whole adjustment being performed in the E. F. system.

At this point, a discussion of what constitutes a short arc may be useful. As pointed out in Brown,[1] the rationale for the short arc approach is that orbital errors resulting from the enforcement of a reasonably accurate set of potential coefficients truncated at fairly low degree and order can, under proper circumstances, be accommodated by slight adjustments of the six state vector parameters. For example, in the case of a nearly circular orbit at an altitude of 1000 km, the integration of an arc of 600 sec using enforced potential coefficients (constants) truncated at $(n, m) = (4, 4)$ can result in errors at the extremities of the arc approaching 1 to 2 m when the state vector at midarc is held to its correct value. However, when the state vector is allowed appropriate freedom to adjust, the error in the resulting arc may be reduced to a level of a few centimeters. In the general context of the short arc method, then, a "short arc" is simply one that is sufficiently short so that positional errors attributable to the errors in enforced potential coefficients (combined with errors of truncation) are "sufficiently well" accommodated over the arc by an adjustment of the state vector. What constitutes "sufficiently well" depends on the character of the particular problem. By virtue of such considerations, it can be said that the short arc method, by definition, does not require that specific attention be paid to errors in potential coefficients insofar as each satellite arc is concerned. This is compatible with the functional relationship appearing in equation (43) of Brown.[2] In particular, the coordinates of a satellite point on a short arc are expressed as independent of the adjustable potential coefficients.

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1. Brown, D.C. (1967) Review of Current Geodetic Satellite Programs and Recommendations for Future Programs, Report for NASA Headquarters, Contract No. NASW-1469.

2. Brown, D.C. (1973) Investigation of the Feasibility of a Short Arc Reduction of Satellite Altimetry for Determination of the Oceanic Geoid, Report No. AFCRL-TR-73-0520, Air Force Cambridge Research Laboratories (LW), Bedford, MA.

The state vector components for a short arc adjustment often contain standard errors which serve subsequently for weighting purposes. Weighting must be independent for each arc, Blaha[3]. Weights for orbital parameters, although weak, are required in a short arc satellite altimetric adjustment; without weights the determination of the six state vector parameters would be impossible since altimetric measurements contain little or no information about the spatial orientation of the orbital plane. However, the short arc approach to the utilization of satellite altimetry is not dependent on intensive external tracking and can be visualized as follows:

(1) Reference orbits approximately accurate to 20 m (rms) are initially established from routine global tracking (VHF Doppler tracking alone is sufficient to establish suitable reference orbits).

(2) The reference orbits are divided into a large number of subarcs situated over oceanic regions; such subarcs are limited in length to at most 1/4 revolution and average about 1/6 revolution.

(3) Each subarc is treated as an independent orbit with the epoch at midarc having a state vector subject to a-priori weighting consistent with the quality of the reference orbit.

(4) For each subarc, observation equations arising from satellite altimetry are introduced (such equations, of course, introduce a model representing the oceanic geoid).

(5) The adjustment attempts the simultaneous recovery of all geoidal parameters along with revised estimates of the state vectors of all orbital arcs (which may number in the thousands for dense global coverage).

GEOS-3 altimetric observations over a portion of the North Atlantic were provided to AFGL by NASA; another set of altimetric observations over adjacent portions of the Indian and South Pacific Oceans was provided by the Naval Surface Weapons Center (NSWC). Figure 1 depicts one-hundred and twelve passes of GEOS-3 NASA data used in the North Atlantic regions, and one-hundred and eleven passes of GEOS-3 NSWC data used in the Indian and Pacific Oceans area.

During the initial preprocessing of the GEOS-3 data, certain characteristics were identified for editing criteria. The first editing level automatically examined altimetic measurements for gross errors that occurred from data handling procedures, e.g., tape parity errors and measurement identification problems. The second editing procedure was to examine the altimetic measurements for continuity

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3. Blaha, G. (1975) The Combination of Gravity and Satellite Altimetry Data for Determining the Geoid Surface, Report of DBA Systems, Inc.; AFCRL Report No. 75-0347, Air Force Cambridge Research Laboratories, Hanscom AFB, MA.

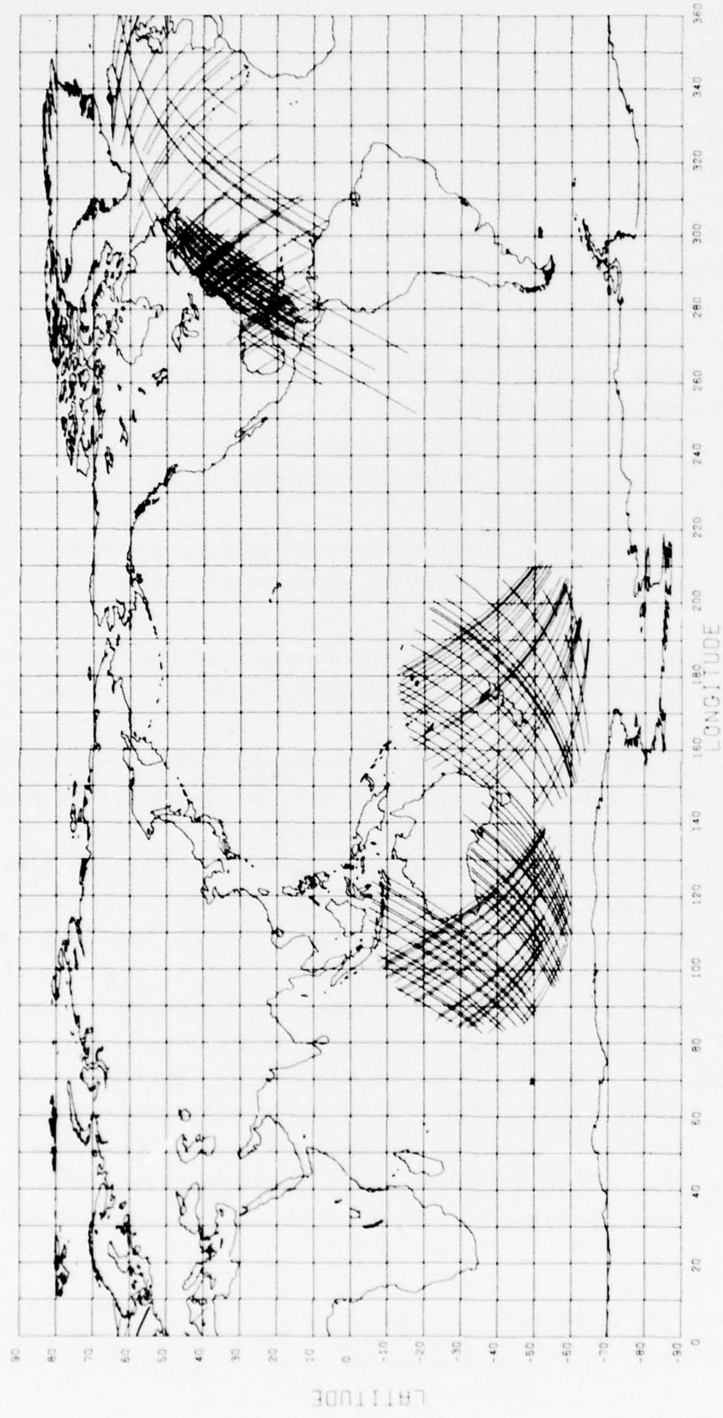

Figure 1. GEOS-3 Ground Track

and to eliminate abrupt point-to-point changes. The final editing is based on a three sigma criteria when compared to a polynomial smoothing function.

The positional a-priori standard errors of the state vectors were assumed to be 24 m, 17 m, and 8 m in the in-track, cross-track, and up-track directions, respectively. The velocity standard errors were similarly assumed to be 0.02 m/sec, 0.02 m/sec, and 0.01 m/sec. The a-priori standard error was assumed to be 1.0 m for all altimetric measurements. The anomaly values, positional information, and the a-priori standard error of each gravity anomaly was read from a tape supplied to AFGL by by Defense Mapping Agency Aeronautical Center (DMAAC).

2. ADJUSTMENT MODELS

2.1 Satellite Altimetry Model

The geometry of this model can be symbolized by the vector equation

$$\vec{H} = \vec{R} - \vec{r}$$

as indicated in Figure 2, where H, R, and r are the magnitudes of the above vectors. The letters R and r represent the adjusted quantities that fulfill exactly the prescribed mathematical model. The approximate values of these quantities based on some initial values of parameters will be denoted as R^o and r^o. The adjusted and observed satellite altimetry values will be denoted as H and H^b, respectively. These notations will serve only when forming the "discrepancy" terms (sometimes called constant terms) in observation equations; in other instances, especially when forming the partial derivatives, the indices are dropped. Thus the state vector components for an arc are symbolized by $X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}$ and the coordinates of S, a point on this arc, by X_s, Y_s, Z_s (all in the E.F. system). The epoch (of the state vector) is denoted by t_o and the time of the "event S" is t_s. The position of S for a given $t_s - t_o$ is a function of the state vector, written as

$$X_s = X_s(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}) \quad ,$$

$$Y_s = Y_s(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}) \quad ,$$

$$Z_s = Z_s(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}) \quad .$$

In agreement with the earlier discussion, this position is considered to be independent of the adjustable potential coefficients r_o, C's, and S's.

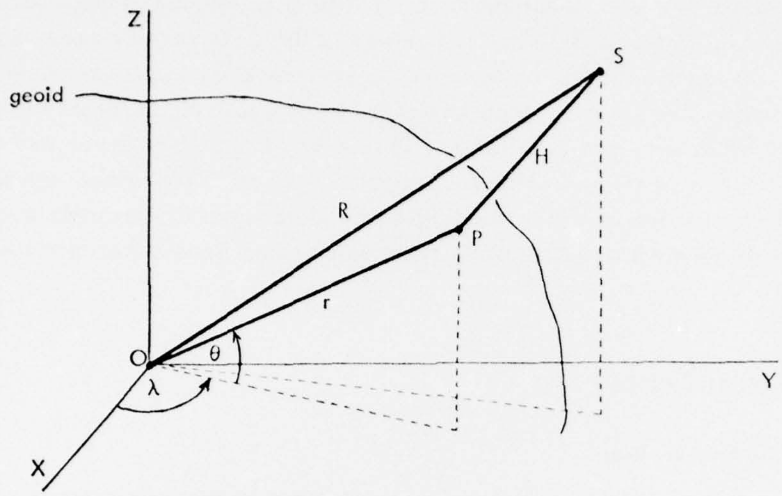

Figure 2. Geometry of Satellite Altimetry in the Earth Fixed (Geocentric) Coordinate System

In order to express r as a function of potential coefficients, the values of θ, λ must be known; they can be computed from the position of P which, in turn, can be obtained from the position of S and from the measured value H^b. However, the direction of $\vec{H}$ is unknown and will be approximated through the use of an ellipsoid. In many respects, the best approximation to the geoid is the geocentric mean earth ellipsoid; the semimajor axis (a) and the eccentricity (e) of this ellipsoid offer the means to approximate the direction of $\vec{H}$ by the geodetic latitude and longitude, ϕ and λ (under the present simplification, OPS in Figure 2 would lie in the meridian plane whose longitude is λ). The longitude is computed from

$$tg\lambda = Y_s/X_s \quad ,$$

while the latitude may be obtained through an iterative process as

$$tg\phi = [Z_s/(X_s^2 + Y_s^2)^{1/2}]\,(N + h)/[N(1 - e^2) + h] \quad ,$$

$$N = a/(1 - e^2 \sin^2 \phi)^{1/2} \quad ,$$

$$h = (X_s^2 + Y_s^2)^{1/2}/\cos\phi - N \quad .$$

The position of P is thus computed to a good approximation as follows:

$$X_p = X_s - H^b \cos\phi \cos\lambda \quad ,$$

$$Y_p = Y_s - H^b \cos\phi \sin\lambda \quad ,$$

$$Z_p = Z_s - H^b \sin\phi \quad ;$$

this yields θ from the relation

$$tg\theta = Z_p/(X_p^2 + Y_p^2)^{1/2} \quad .$$

Both θ and λ are seen to depend on the state vector parameters through the position of S,

$$\theta = \theta(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}) \quad , \tag{1a}$$

$$\lambda = \lambda(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}) \quad . \tag{1b}$$

The values of R and r can now be expressed as functions of the adjustable parameters, namely

$$R = (X_s^2 + Y_s^2 + Z_s^2)^{1/2} \tag{2}$$

and, from Blaha,[3] equation (46a),

$$r = r_o \left[1 + \sum_{n=2}^{\infty} (a/r)^n \sum_{m=0}^{n} (C_{nm} \cos m\lambda + S_{nm} \sin m\lambda) P_{nm} (\sin \overline{\theta}) \right]$$

$$+ \frac{1}{2} \omega^2 r_o r^3 \cos^2 \overline{\theta}/(kM) \tag{3}$$

where ω is the angular velocity of earth's rotation and kM is the product of the gravitational constant and the earth's mass; r_o, C_{nm} and S_{nm} were already labeled as r_o, C's and S's, while P_{nm} $(\sin \theta)$ are the conventional Legendre functions. In practice, the series expansion in (3) is truncated at some suitable n = N. The parameter r_o corresponds to the radius of a fictitious spherical earth having the same potential as the geoid, namely

$$r_o = kM/W_o \quad ,$$

where W_o is the gravity potential of the geoid. To obtain a starting value of r_o in case it is subject to adjustment, one could use U_o, the potential of the mean earth ellipsoid. From (2) and (3) together with equations (1) it follows that

$$R = R\,(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}) \quad , \tag{2'}$$

$$r = r\,(r_o, C's, S's; X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}) \quad . \tag{3'}$$

It may be reiterated that although r is principally a function of the potential coefficients, it becomes dependent on the orbital elements through the altimeter measurement which is considered perpendicular to the geoid or, to a good approximation, perpendicular to the ellipsoid.

Upon using the approximate values of parameters and the measured altimetry values, the discrepancy terms (L) may be computed in one of the two equivalent formulas below:

$$L = (X_p^2 + Y_p^2 + Z_p^2)^{1/2} - r^o \tag{4a}$$

or

$$L = (R^o - r^o + d) - H^b \quad . \tag{4b}$$

We recall that H is the distance from a satellite to the geoid (and not to the ellipsoid) and that the ellipsoid serves only to approximate the direction along which this distance is measured in order to compute d. The error in d caused by this approximation would amount to less than 4 cm even if the deflection of the vertical were 1' (for simplicity, one can take $H \simeq 1000$ km). Considering the overall precision of satellite altimetry, such errors are negligible. It can be shown that

$$d = \frac{1}{2}\, r\,\delta^2\, (r + H)/H \quad ,$$

where

$$\delta = \bar{\phi} - \theta \quad ,$$

$\bar{\phi}$ being the geocentric latitude of S and θ representing the geocentric latitude of P as in Figure 2. In particular, $\bar{\phi}$ is computed from

$$tg\bar{\phi} = Z_s/(X_s^2 + Y_s^2)^{1/2} \quad .$$

For some purposes, the quantity d could be roughly evaluated as

$$d \simeq 4.8 \text{ m} \sin^2 \theta \ldots \text{ per 1000 km of H} \quad .$$

For the partial differentation, the model

$$H \simeq R - r$$

may be shown to be entirely sufficient. In the most general form, the observation equations are written as

$$V = \frac{\partial(R - r)}{\partial\,(\text{parameters})}\, d\,(\text{parameters}) + L \quad .$$

Upon considering (2'), (3') this is

$$V = [\partial R/\partial(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z})] \begin{bmatrix} dX \\ dY \\ dZ \\ d\dot{X} \\ d\dot{Y} \\ d\dot{Z} \end{bmatrix} - [\partial r/\partial(r_o, C's, S's)] \begin{bmatrix} dr_o \\ dC's \\ dS's \end{bmatrix}$$

$$- [\partial r/\partial(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z})] \begin{bmatrix} dX \\ dY \\ dZ \\ d\dot{X} \\ d\dot{Y} \\ d\dot{Z} \end{bmatrix} + L \quad , \tag{5}$$

where the differentiation of R with respect to the state vector components is done via the chain rule applied to matrices as follows:

$$\frac{\partial R}{\partial(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z})} = \frac{\partial R}{\partial(X_s, Y_s, Z_s)} \frac{\partial(X_s, Y_s, Z_s)}{\partial(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z})} \quad .$$

With the aid of (2), the first matrix on the right hand side is seen to be composed of the three directional cosines. The second (3×6) matrix, denoted $\tilde{M}_1$, is given as

$$\tilde{M}_1 = \frac{\partial(X_s, Y_s, Z_s)}{\partial(x_s, y_s, z_s)} \frac{\partial(x_s, y_s, z_s)}{\partial(x, y, z, \dot{x}, \dot{y}, \dot{z})} \frac{\partial(x, y, z, \dot{x}, \dot{y}, \dot{z})}{\partial(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z})} \quad .$$

Here, the lower case letters are equivalent to their counterparts (capital letters), except that they refer to a chosen inertial system. The first (3×3) matrix on the right-hand side of the last equation represents a rotation and the second (3×6) matrix consists of the first three rows of the matrizant. Upon using the advantageous choice of the inertial system for each arc so that it coincides with the E. P. system at the epoch (t_o), one can express

$$\frac{\partial(X_s, Y_s, Z_s)}{\partial(x_s, y_s, z_s)} = \begin{bmatrix} \cos[\omega(t_s - t_o)] & \sin[\omega(t_s - t_o)] & 0 \\ -\sin[\omega(t_s - t_o)] & \cos[\omega(t_s - t_o)] & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and

$$\frac{\partial(x, y, z, \dot{x}, \dot{y}, \dot{z})}{\partial(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z})} = \begin{bmatrix} I & 0 \\ T & I \end{bmatrix} , \quad T = \begin{bmatrix} 0 & -\omega & 0 \\ \omega & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} .$$

the partial derivatives of R are thus written as

$$\partial R/\partial(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}) = [X_s/R, \; Y_s/R, \; Z_s/R] \tilde{M}_1 \quad . \tag{6}$$

From (3), the partial derivatives $\partial r/\partial(r_o, C's, S's)$ can be expressed in the following manner:

$$\partial r/\partial r_o = (1 + P_1 + \frac{1}{2} D)/Q \quad , \tag{7a}$$

$$\partial r/\partial C_{no} = r_o(a/r)^n \, P_n(\sin\theta)/Q \quad , \tag{7b}$$

$$\left.\begin{aligned} \partial r/\partial C_{nm} &= r_o(a/r)^n \, P_{nm}(\sin\theta) \cos m\lambda/Q \quad , & (7c) \\ \partial r/\partial S_{nm} &= r_o(a/r)^n \, P_{nm}(\sin\theta) \sin m\lambda/Q \quad , & (7d) \end{aligned}\right\} \; m > 0$$

where

$$Q = 1 + (r_o/r)(P_2 - 3D/2) \quad ,$$

$$P_1 = \sum_{n=2}^{\infty} (a/r)^n \, S(n) \quad , \quad P_2 = \sum_{n=2}^{\infty} n(a/r)^n \, S(n) \quad ,$$

$$S(n) = \sum_{m=0}^{n} (C_{nm} \cos m\lambda + S_{nm} \sin m\lambda) P_{nm} (\sin \theta) \quad ,$$

$$D = \omega^2 r \cos^2 \theta / C \quad , \quad C = kM/r^2 \quad .$$

If the quantity D is replaced by zero in all of the above formulas, one recovers equations (49) of Blaha,[3] where the effect of the earth rotation was not included. With the present notations, equation (3) reads

$$r = r_o(1 + P_1 + \frac{1}{2} D) \quad ;$$

the term $(1 + P_1 + \frac{1}{2} D)$ already appeared in (7a).

The differentiation of r with respect to the state vector parameters can be accomplished along the following lines:

$$-\partial r/\partial(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}) \simeq -(\partial r/\partial\phi)(\partial\phi/\partial\theta)(\partial\theta/\partial\overline{\phi})[\partial\overline{\phi}/\partial(X_s, Y_s, Z_s)]$$
$$\times [\partial(X_s, Y_s, Z_s)/\partial(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z})] \quad ,$$

where the last (3×6) matrix had been denoted as $\tilde{M}_1$ and where the ellipsoid approximation to the geoid has been used for this purpose $(\partial r/\partial\lambda = 0)$. According to Blaha,[4] this expression may be developed to read

$$-\partial r/\partial(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}) \simeq a\, e^2 \sin \overline{\phi} \cos \overline{\phi} [\partial\overline{\phi}/\partial(X_s, Y_s, Z_s)] \tilde{M}_1 \quad ,$$

and finally,

$$-\partial r/\partial(X, Y, Z, \dot{X}, \dot{Y}, \dot{Z}) \simeq (a/R) e^2 \, [-(X_s/R) \sin^2 \overline{\phi}, \; -(Y_s/R) \sin^2 \overline{\phi},$$
$$(Z_s/R) \cos^2 \overline{\phi}] \tilde{M}_1 \quad . \tag{8}$$

4. Blaha, G. (1977) Refinement of the short arc satellite altimetry adjustment model, paper published in Bulletin Geodesique.

Equations (6) and (8) may now be substituted into (5) so that the form of observation equations is the following:

$$V = \{(X_s/R)[1 - (a/R)\, e^2 \sin^2 \bar{\phi}] \quad , \quad (Y_s/R)[1 - (a/R)\, e^2 \sin^2 \bar{\phi}] \quad ,$$

$$(Z_s/R)[1 + (a/R)\, e^2 \cos^2 \bar{\phi}]\} \times \tilde{M}_1 \begin{bmatrix} dX \\ dY \\ dZ \\ d\dot{X} \\ d\dot{Y} \\ d\dot{Z} \end{bmatrix}$$

$$- [\partial r/\partial(r_o, C's, S's)] \begin{bmatrix} dr_o \\ dC's \\ dS's \end{bmatrix} + L \quad , \tag{9}$$

where L was given in (4a) or (4b) and where $\partial r/\partial(r_o, C's, S's)$ appear explicitly in (7). For each satellite arc, a new set of (weighted) state vector parameters is introduced. The adjustment (of the satellite altimetry alone or in combination with the gravity-type data) is effectuated by the method described in Chapter 2 of Blaha.[3]

The values $-(a/R)\, e^2 \sin^2 \bar{\phi}$, $-(a/R)\, e^2 \sin^2 \bar{\phi}$, $+(a/R)\, e^2 \cos^2 \bar{\phi}$ in equation (9) may be called "refinement of the partial derivatives" as far as the state vector parameters are concerned. These corrections can change the "unrefined" partial derivatives up to 0.6 percent of their values (the average correction is roughly 0.3 percent). If the corrections were not applied, the partial derivatives with respect to the state vector would often be accurate to only 2 to 3 significant digits. By including them, the validity of these partial derivatives is extended to another 2 or more significant digits. The remaining errors stemming from the ellipsoidal approximation to the geoid are believed to be smaller than the above corrections by at least one order of magnitude and they diminish with decreasing deflections of the vertical. Furthermore, such remaining errors are random in nature unlike the above corrections. The practical usefulness of the refinement in the partial derivatives may be illustrated as follows. A small set of satellite altimetry data was adjusted by the AFGL program SARRA (Short Arc Reduction of Radar Altimetry) described in Hadgigeorge and Trotter.[5] In this program, a different set of coefficients from C's and S's is used, but the state vector parameters are treated exactly as described earlier. After two iterations with the "unrefined" partial derivatives, the parameters received no further

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5. Hadgigeorge, G., and Trotter, J.E. (1976) Short arc reductions of GEOS-3 altimetry data, Paper presented at the AGU Fall Meeting.

corrections. However, using the refined partial derivatives, already the first adjustment yielded all the final values. In this way iterations may be eliminated altogether, which results in important savings in terms of computer run-time.

2.2 Gravity Anomaly Model

The gravity anomaly model used in the present reductions corresponds closely to that given by Rapp,[6] equation (3).

$$\Delta g \simeq (kM/r^2) \sum_{n=2}^{\infty} (n-1)(a/r)^n \sum_{m=0}^{n} (\Delta C_{nm} \cos m\lambda + \Delta S_{nm} \sin m\lambda) P_{nm} (\sin \bar{\phi}) \; , \tag{10}$$

where $\bar{\phi}$ is the geocentric latitude of the point associated with Δg. Further,

$$\Delta C_{20} = C_{20} - C^*_{20} \; ,$$

$$\Delta C_{40} = C_{40} - C^*_{40} \; ,$$

$$\Delta C_{60} = C_{60} - C^*_{60} \; ,$$

the other ΔC's *and* ΔS's *being essentially* the conventional spherical harmonic potential coefficients (C's and S's) themselves. The values C^*_{20}, C^*_{40}, and C^*_{60} refer to the equipotential ellipsoid of revolution adopted as a base reference surface in the normal gravity field. The impact of replacing the coefficients C^*_{80}, etc., by zeros is negligible. The maximum possible contribution of $C^*_{60} \simeq -0.000\,000\,0061$ in equation (10) represents only 0.03 mgal. The contribution of C^*_{80} would be two orders of magnitude smaller and a similar pattern would hold for further coefficients. By comparison, the maximum contribution of $C^*_{40} \simeq +0.000\,0024$ is 7 mgal.

In considering r_o to be an adjustable parameter as in the preceding section, the model (10) can be rewritten as

$$\Delta g \simeq C \left[-2(r/r_o^{*2})\, \Delta r_o + \sum_{n=2}^{\infty} (n-1)(a/r)^n \, \Delta S(n) \right] \; , \tag{11}$$

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6. Rapp, R.H. (1972) Improved Models for Anomaly Computations from Potential Coefficients, Department of Geodetic Science, Report No. 181, The Ohio State University, Columbus.

where

$$\Delta r_o = r_o - r_o^* \quad ,$$

$$r_o^* = kM/U_o \quad ,$$

U_o being the potential of the reference ellipsoid: in practice, the initial value of Δr_o is zero and the parameter r_o is often weighted. The remaining new notation is

$$\Delta S(n) = \sum_{m=0}^{n} (\Delta C_{nm} \cos m\lambda + \Delta S_{nm} \sin m\lambda) \, P_{nm} (\sin \bar{\phi}) \quad .$$

The series inside the brackets in (11) may be expressed as

$$\sum_{n=2}^{\infty} (n-1)(a/r)^n \Delta S(n) = \sum_{n=2}^{\infty} (n-1)(a/r)^n S(n) - (a/r)^2 C_{20}^* P_2 (\sin \bar{\phi})$$

$$- 3 (a/r)^4 C_{40}^* P_4 (\sin \bar{\phi}) - 5 (a/r)^6 C_{60}^* P_6 (\sin \bar{\phi}) \quad .$$

The discrepancy terms are obtained by the usual approach as

$$L = \Delta g - \Delta g^b \quad , \tag{12}$$

where Δg is a function of the parameters to be adjusted (see equation 11) and Δg^b is the input - or "observed" - value of the gravity anomaly. When the gravity anomaly model is differentiated with respect to the parameters r_o, C's and S's, the value of r may be considered constant. The observation equations have the following form:

$$V = [\partial \Delta g / \partial (r_o, C\text{'s}, S\text{'s})] \begin{bmatrix} dr_o \\ dC\text{'s} \\ dS\text{'s} \end{bmatrix} + L \quad , \tag{13}$$

where, from (11),

$$\partial\Delta g/\partial r_o = -2(r/r_o^{*2}) \quad , \tag{14a}$$

$$\partial\Delta g/\partial C_{no} = (n-1)(a/r)^n P_n(\sin\bar{\phi}) \quad , \tag{14b}$$

$$\left.\begin{aligned} \partial\Delta g/\partial C_{nm} &= (n-1)(a/r)^n P_{nm}(\sin\bar{\phi})\cos m\lambda \quad , \qquad (14c) \\ \partial\Delta g/\partial S_{nm} &= (n-1)(a/r)^n P_{nm}(\sin\bar{\phi})\sin m\lambda \quad . \qquad (14d) \end{aligned}\right\} \quad m > 0$$

2.3 Error Propagation

Either of the two kinds of observation equations described in the previous sections may be adjusted separately if the data is sufficient. However, the most useful results are obtained from their combination; the reader is again referred to Chapter 2 of Blaha,[3] for the algorithms that make the combination and the solution of all the parameters possible (including the state vector parameters for each satellite arc). After the adjustment, the variance-covariance matrix of r_o, C's, and S's becomes available (denoted Σ r_o, C's, S's). This matrix is instrumental in computing the a-posteriori variances of r and Δg at chosen geographic locations. In the first case, the values of $\partial r/\partial(r_o$, C's, S's) are utilized and in the *second case*, the values of $\partial\Delta g/\partial(r_o$, C's, S's) are needed. Both kinds of partial derivations have been presented explicitly in equations (7) and (14), respectively. The formulas giving either variance are of course unchanged whether a separate adjustment or a combined adjustment has been performed. They are used to generate a grid of a-posteriori standard derivations in geoid undulations and in gravity anomalies. Such grids may serve to construct a contour map of standard derivations; similar grids lead to contour maps of geoid undulations and gravity anomalies themselves.

The value of r is closely related to the geoid undulation (N) as follows:

$$N \simeq N' = r - r' \quad , \tag{15}$$

where r' is the radial distance to the reference ellipsoid depicted in Figure 3. It can be shown that for $|N|$ as large as 120 m, the error in (15) is still negligible; in particular,

$$|N' - N| < 0.7 \text{ mm} \quad .$$

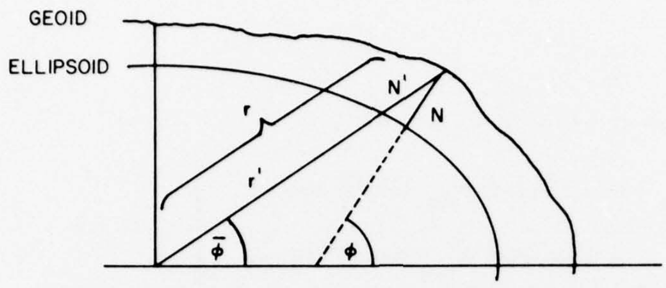

Figure 3. Relation of the Value of r With That of the Geoid Undulation (N)

Since r' is constant (independent of the adjustable parameters), the variance of N is the same as the variance of r; it is obtained from the law of propagation of variances-covariances, namely

$$\sigma_N^2 = \sigma_r^2 = [\partial r/\partial(r_o, C's, S's)]\, \Sigma_{r_o, C's, S's}\, [\partial r/\partial(r_o, C's, S's)]^T \quad . \tag{16}$$

The gravity on the geoid is by definition

$$g = \Delta g + \gamma \tag{17}$$

and the pertinent variances similarly are

$$\sigma_g^2 = \sigma_{\Delta g}^2 = [\partial \Delta g/\partial(r_o, C's, S's)]\, \Sigma_{r_o, C's, S's}\, [\partial \Delta g/\partial(r_o, C's, S's)]^T \quad , \tag{18}$$

γ being the normal gravity (independent of the adjustable parameters).

3. NUMERICAL RESULTS

A combined global adjustment of satellite altimetry data and gravity anomaly data has been performed as described in the Introduction of Section 1. The adjustment model that has served in this task is characterized by the presence of spherical harmonic potential coefficients up to and including the degree and order (14, 14). The option to include a-priori standard errors in the coefficients has been exercised in the following manner:

$$\sigma_{r_o} = 1 \text{ m} ,$$

$$\sigma_{C_{20}} \simeq 1.1 \times 10^{-6} \quad (\simeq 0.001|C_{20}|) ,$$

$$\sigma_{C_{22}} \simeq 1.6 \times 10^{-7} \quad (\simeq 0.1|C_{22}|) ,$$

$$\sigma_{S_{22}} \simeq 1.0 \times 10^{-7} \quad (\simeq 0.1|S_{22}|) .$$

The five "forbidden" coefficients have been held fixed at the zero value. Due to the scarcity of satellite altimetry data, a separate altimetry adjustment in the (14, 14) geopotential model failed (the system of normal equations was severely ill-conditioned). The gravity anomaly data has been adjusted alone and in combination with the altimetry data. The present gravity anomaly data also exhibits gaps; furthermore, in several areas the a-priori standard deviation is extremely large. This is locally reflected by relatively large a-posteriori standard errors in geoid undulations and gravity anomalies in the separate gravity anomaly adjustment as well as in the combined adjustment.

The inclusion of satellite altimetric data has significantly improved the external reliability of the gravity anomaly adjustment. This is due not only to the addition of data in areas where it has been previously lacking, but also to the fact that a completely different and independent source of data has been allowed to play its role in the determination of the earth's gravity field. As far as the internal precision is concerned, significant improvements have been noted, especially in the areas of the thus added data. This precision is represented by a-posteriori standard errors in geoid undulations and gravity anomalies. As an example, the standard error in geoid undulation in the Indian Ocean region (around $\phi = -60^{\circ}$ and $\lambda = 120^{\circ}$) improved more than 10 times (from 8.3 m to 0.82 m). The internal precision is of course invariably too optimistic with regard to the "real world" since it reflects the assumption that the geoid is exactly represented by the spherical harmonic expansion truncated at a given degree and order (any further coefficients as well as their standard errors are considered to be exactly zero). However, the a-posteriori standard errors can be of great assistance in determining the goodness of fit accomplished by a given reduction, in depicting areas of insufficient data, etc.

The combined adjustment in the (14, 14) geopotential model yielded the values of geoid undulations (N), gravity anomalies (Δg), a-posteriori standard errors in N(σ_N), and a-posteriori standard errors in Δg ($\sigma_{\Delta g}$), computed all in a $10^{\circ} \times 10^{\circ}$ geographic grid. These values have been subsequently used for plotting contour maps. Figures 4 and 5 show the contour maps for N (contour interval is 10 m)

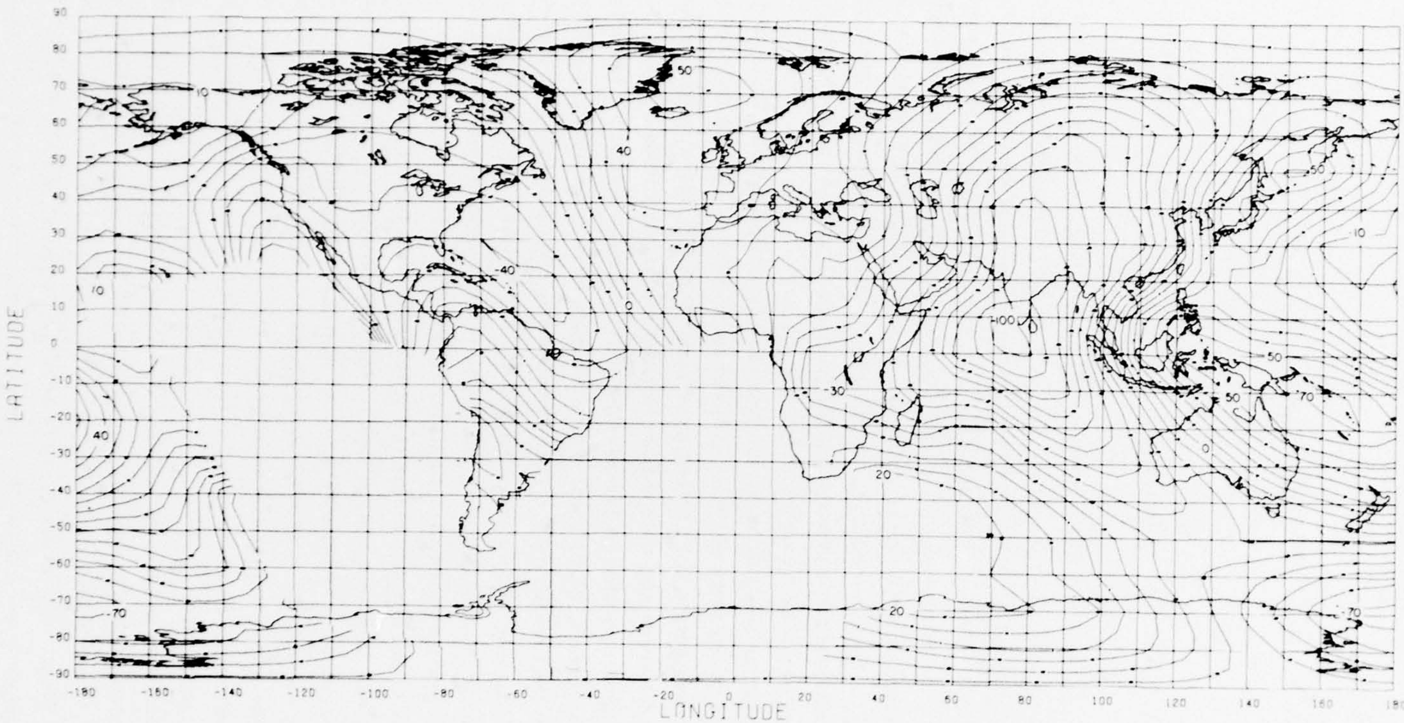

Figure 4. GEOID Height Contours. Contour interval = 10 m; earth radius = 6378.140 km; 1/f = 298.257; gm = 398600.5 km^3/sec^2

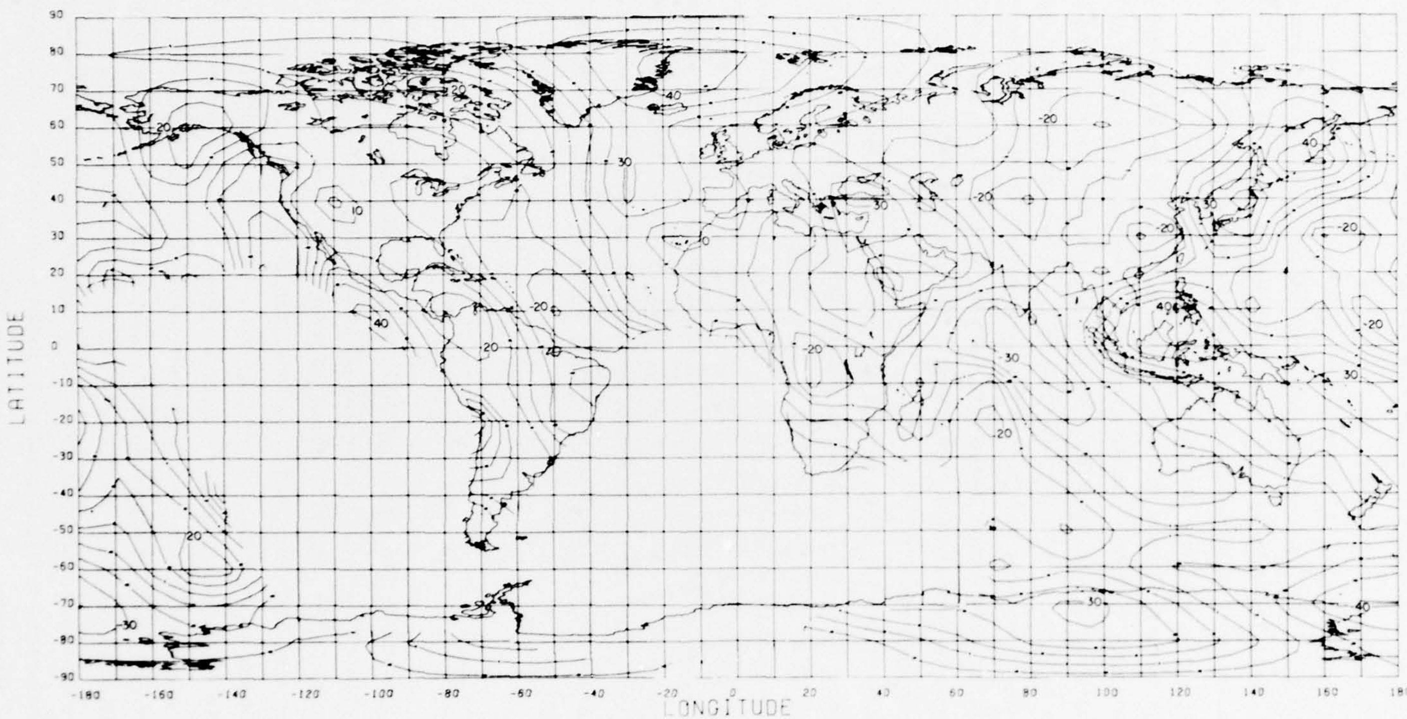

Figure 5. Gravity Anomaly Contours. Contour interval = 10 mgal

and Δg (contour interval is 10 mgal), respectively. The areas of insufficient data on these maps have been blanked out. The "cut-off" decision for this purpose has been facilitated precisely by the contour maps of standard errors. A certain contour (in this case the 3 m contour) has been chosen from the plot associated with σ_N beyond which the data has been deemed insufficient. The values of σ_N in the areas of week geoid determination rise sharply and are three to ten fold larger than the values in all the other parts of the globe.

The geoid contour map has been compared with the geoidal maps appearing in Hadgigeorge and Trotter.[5] In the areas of comparison, the shape of the geoid exhibits remarkable similarities. In addition, the numerical values of geoid undulations themselves agree very satisfactorily. It is to be noted that the geoid presented in this reference had in turn been compared, along four different profiles, with a detailed gravimetric geoid (see Sections 3.2 and 3.3); the results of these comparisons were reported as excellent. The present reduction is thus verified from independent sources.

In order to provide more insight into the interdependence of the degree and order of truncation and the detail in geoidal features, a new adjustment has been performed, this time in conjunction with a (13, 13) geopotential model. This has yielded a new set of values N, Δg, σ_N, and $\sigma_{\Delta g}$. The pertinent contour maps for N and Δg appear respectively in Figures 6 and 7, in which the same areas have been blanked out as previously. As expected, somewhat lower standard errors have been noticed compared to their counterparts in the (14, 14) geopotential model. From the last two figures, a certain amount of smoothing may be observed if the contours are compared to these of the two preceding figures; this fact, too, was to be expected.

4. CONCLUSIONS

A simultaneous adjustment of real satellite altimetric and gravity anomaly data in conjunction with a (14, 14) geopotential model appears to have been accomplished satisfactorily. The geoidal features extracted from this reduction reveal a good agreement with those obtained from independent sources. A comparison of internal precision has demonstrated the beneficial effect of adding altimetric data to the existing body of gravity anomaly data. However, adding this relatively new and completely independent source to a more traditional type of data in a combined adjustment is beneficial also for other reasons, such as increasing the external reliability of the results. An important fact that has been confirmed during the real data reductions — and not only in some computer simulations — is that highly accurate reference orbits are not a stringent requirement in the short arc mode of satellite altimetry.

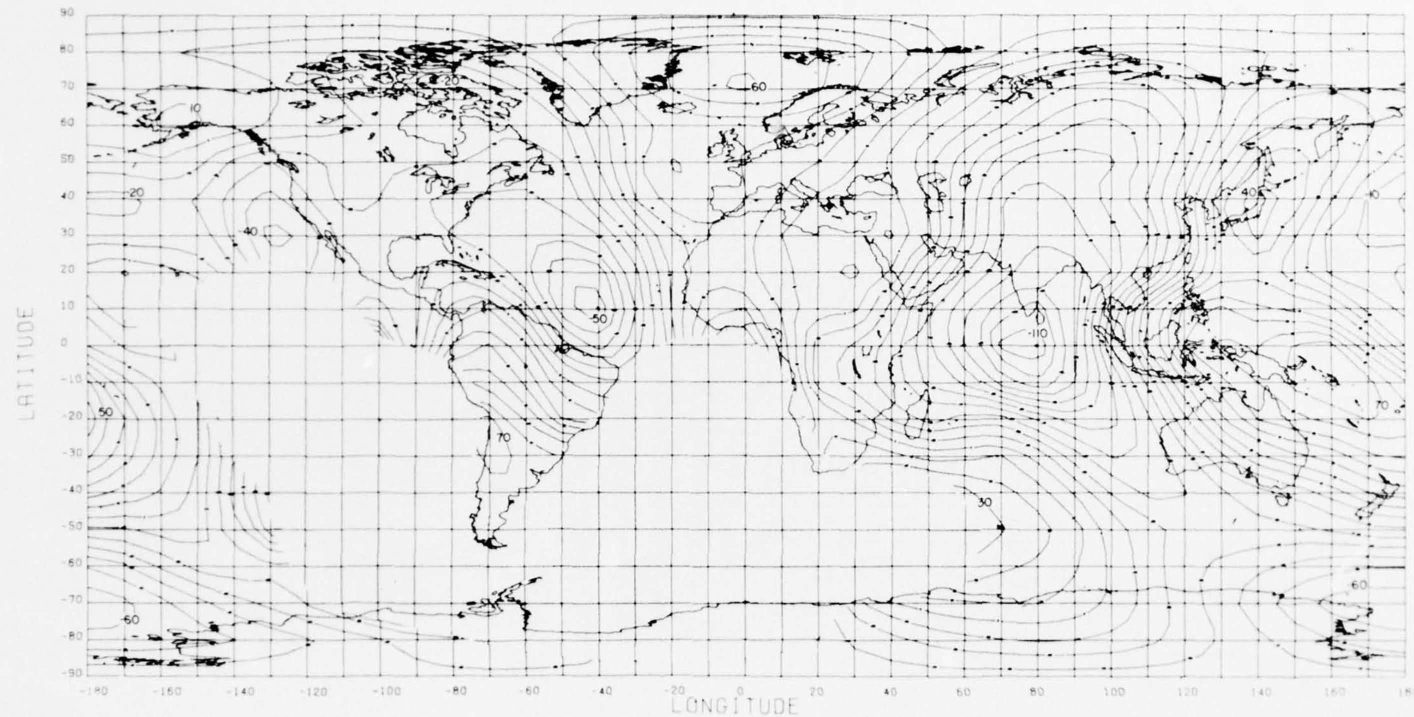

Figure 6. GEOID Height Contours. Contour Interval = 10 m; earth radius = 6378.140 km; 1/f = 298.257; gm = 398600.5 km^3/sec^2

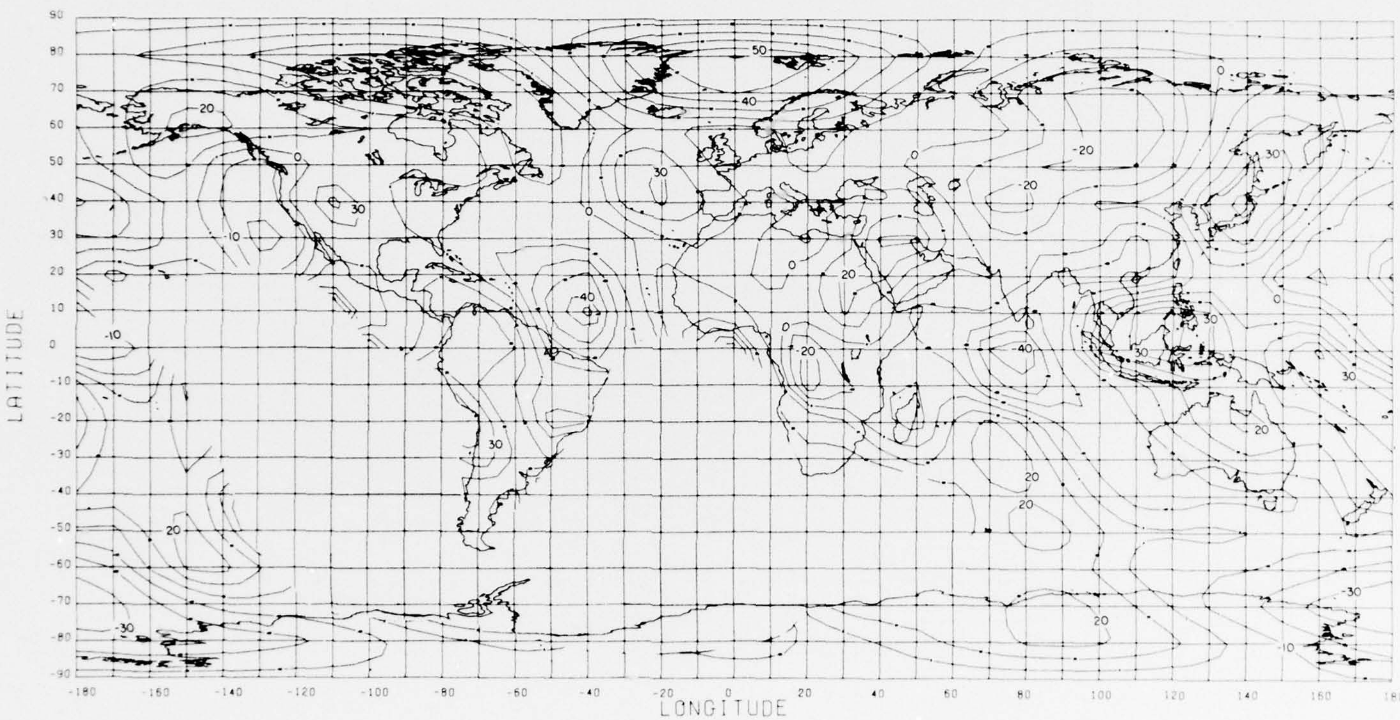

Figure 7. Gravity Anomaly Contours. Contour interval = 10 mgal

The present reduction in which only a part of the existing satellite altimetric and gravity anomaly data has been utilized indicate that the global geoid recovery with a standard error better than 1 m could be achieved without undue difficulty. The outcome of this study also points to the desirability of further refining the program SAGG so that more detailed geoidal features in the regions with abundant data may be extracted without substantially increasing the computer core-space and run-time requirements.

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